Author's Solution

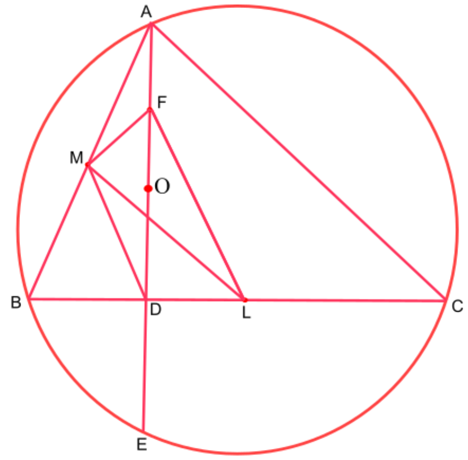
To Prove :

$\angle LFM = \angle B$

Construction :

Join DM & LM.

Fix a point O on AD such that OD = DE.



Proof:

DF = $\frac{1}{2}AE$ (given)

By construction, OD = DE

⟹ AF = FO -------------------------------(1)

⟹ By the result on Orthocentre available in the book Novelties of Geometry (Novelty 3 in page 15 & 16) uploaded in this website,

O is the orthocentre ---------------------------(2)

AD is an altitude of ΔABC.

⟹ AD ⊥ BC ---------------------------------(3)

∴ M, the midpoint of AB is the circumcentre of ΔABD.

⟹MD = MA

⟹ $\angle MAD = \angle MDA$ ---------------------------(4)

As per the Nine Point Circle Theorem, the nine points circle of a Δ passes through

1. the feet of its altitudes
2. the midpoints of its sides and
3. the midpoints of the line segments joining the vertices and the orthocentre.

By (1) & (2),

the nine points circle of ΔABC will pass through M,D,L & F.

⟹ MDLF is concyclic

⟹ $\angle MDF = \angle MLF$ --------------------------------(5)

And $\angle LDF = \angle LMF = 90°$ -----------------------(6)

(4) & (5) ⟹ $\angle MAD = \angle MLF$ --------------------- (7)

Now, in ΔABD & ΔLFM $\angle BAD = \angle MLF$ [from (7) above]

$\angle ADB = \angle LMF = 90°$ [from (6) above]

∴ ΔABD ~ΔLFM

⟹ $\angle LFM = \angle B$ -------------------Proved

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